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COMMENT

Two-fluid picture of the SK model of a spin glass

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Abstract. We show how the known properties of the SK model of a spin glass fit naturally into a two-fluid description. The normal fluid is responsible for thermal excitations, whereas the condensed part yields a temperature independent zero field susceptibility at all temperatures below T_c. The relevance of the overblocking effect to the SK model is also pointed out.

1. Introduction

The Sherrington–Kirkpatrick (SK) model of a spin glass is defined by (Sherrington and Kirkpatrick 1975) the Hamiltonian

$$\mathscr{H} = -\tfrac{1}{2}\sum_{(ij)} J_{ij}\boldsymbol{S}_i \cdot \boldsymbol{S}_j - H\sum_i S_i^1 \qquad i, j = 1, 2, \ldots, N \tag{1}$$

where $\boldsymbol{S}_i$ is an m-dimensional (classical) unit vector, H is the magnetic field (parallel to the 1 direction) and the couplings $\{J_{ij}\}$ between any pair of sites (i, j) are independent quenched random variables, with probability distribution

$$p(J_{ij}) = \left(\frac{N}{2\pi J^2}\right)^{1/2} \exp\left(-\frac{N}{2J^2}\left(J_{ij} - \frac{J_0}{N}\right)^2\right) \tag{2}$$

where the N factors ensure a proper thermodynamic limit when one tries to calculate the quenched free energy:

$$F = -T\int \prod_{(ij)} p(J_{ij})\,\mathrm{d}J_{ij} \ln Z\{J_{ij}\} \tag{3a}$$

$$Z\{J_{ij}\} = \operatorname*{Tr}_{\{S_i\}} \exp(-\beta\mathscr{H}). \tag{3b}$$

The rest of the paper will deal, if not otherwise specified, with the 'pure' Ising case ($m = 1$, $J_0 = 0$). In this case, one can briefly summarise what has become progressively 'clear' since the initial paper (Sherrington and Kirkpatrick 1975):

(i) There is a phase transition at $T_c = J$.

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(ii) Linear response theory fails in the low-field, low-temperature part of the phase diagram (Bray and Moore 1980, Parisi 1980 c).

(iii) The phase transition does not belong to the instability (or Landau) type of phase transitions (De Gennes 1975).

In this note, we focus our attention on the low-temperature properties of the spin glass phase. We first recall what is known or conjectured in § 2. In § 3, we shall study systems with inhomogeneous ground states where frustration and/or demagnetising effects play an important role; we shall in particular stress the importance of the overblocking effect in fully frustrated lattices. In § 4, we interpret spin glass behaviour as that of a two-fluid model, the normal part being blocked (or frustrated) and the condensed part being overblocked.

2. Summary of previous results ($m = 1, J_0 = 0$)

Two main routes have been explored in the SK problem:

(i) Replicas (Edwards and Anderson 1975).

(ii) TAP equations (Thouless *et al* 1977).

Both have their drawbacks and gave misleading results in the past. What is now clear, or at least plausible, is that a conventional order parameter, such as the Edwards and Anderson (EA) order parameter, is not enough to describe the spin glass transition. The TAP equations extremise the free energy with respect to N variables $m_i (m_i = \langle S_i \rangle_T$, where $\langle \ldots \rangle_T$ denotes thermal averaging). Within the framework of the Blandin–Parisi scheme (Blandin 1978, Parisi 1979, 1980a, b, c), the replica approach yields two solutions; one is the SK solution and the other introduces, as an order parameter, a function $q(x)$ such as the one depicted on figure 1. Close to T_c, Parisi's solution has been shown to be marginally stable (Thouless *et al* 1980).

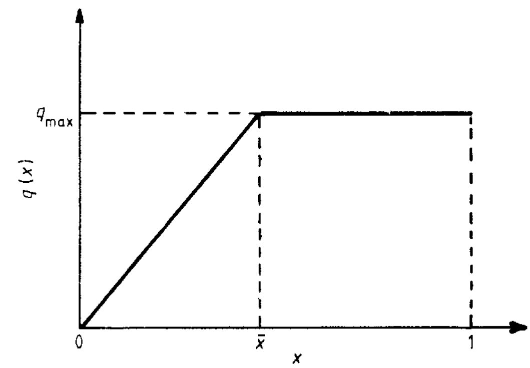

Figure 1. Parisi's order parameter ($H = 0$, $T \leqslant T_c$).

In a magnetic field, the (H, T) plane is divided in two parts by a critical line $H_c(T)$ (De Almeida and Thouless 1978). For $H > H_c(T)$, one only obtains the SK solution. The Parisi–Toulouse (1980) hypothesis assumes that below $H_c(T)$, considered as a second-order line, thermal and magnetic properties are decoupled (failure of linear response theory). In short, it seems that the magnetic field H acts on the $(0 < x < \bar{x})$ part of $q(x)$ (figure 1), whereas thermal fluctuations are connected with the $(\bar{x} < x < 1)$ part.

This fact is consistent with the expression of the zero field magnetic susceptibility χ

$$\chi = \frac{1}{T}\left((1-\bar{x})(1-q_{\mathrm{MAX}}) + \int_0^{\bar{x}} (1-q(x))\,\mathrm{d}x\right) \tag{4}$$

since for T small

$$1 - q_{\mathrm{MAX}} \sim T^2 \tag{5}$$

and studies of the Parisi–Toulouse conjecture (Vannimenus *et al* 1981) lead to

$$\bar{x} = \tfrac{1}{2} \qquad \text{at } T = 0. \tag{6}$$

(Note however that (6) raises the question of a first-order transition across the De Almeida–Thouless curve, a result that has been derived in another context (Sommers 1979).)

The physical picture we shall present below makes use of the overblocking effect for fully frustrated lattices, and of the intermediate state in magnetism and superconductivity, which we now consider.

3. Systems with inhomogeneous ground states

3.1. *The overblocking effect*

Fully frustrated systems (FFS) have been investigated in the literature (Alexander and Pincus 1980, Derrida *et al* 1979). An interesting result for these systems is the overblocking effect (figure 2): it states that one cannot always construct a ground state of a FFS such that all plaquettes are in their minimal energy state. This geometrical effect takes place for space dimension $d > 4$; it comes about because one imposes the constraint of having all plaquettes frustrated and therefore affects Ising as well as vectorial spins. When d goes to infinity, the ground state has an *equal* number of blocked (or frustrated) plaquettes (figure 2(a)) and of overblocked plaquettes (figure 2(b)).

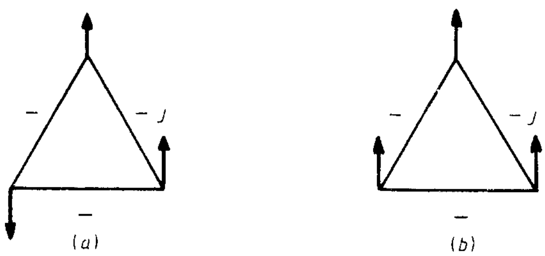

Figure 2. Ising triangular antiferromagnet. The energy per spin is: (a), $-\frac{1}{3}|J|$; (b), $+|J|$.

3.2. *Intermediate state in magnetism and superconductivity*

We shall not attempt to give a thorough description of such systems, but recall that, in some situations, the existence of a demagnetising field gives rise to a nucleation phase transition (De Gennes 1975). Most often, the demagnetising field stems from finite size (or surface) effects, and may induce an instability against the creation of walls. These effects depend on the geometry of the sample.

As long as the external field H is less than a critical value H_c, a magnetic system (e.g. of spherical shape) will have a complicated structure of domains and walls. In a simple approximation, its properties can be decomposed into two parts.

(i) Thermal properties are essentially associated with domains; their magnetisation and entropy per spin depend *upon temperature T but not upon H.* Also, for $H \leqslant H_c$, linear response holds, so that their magnetic susceptibility goes to zero as $T \to 0$.

(ii) Magnetic properties are linked with the walls: as H increases they progressively disappear so that when $H = H_c$ the system behaves as a single domain. For $H \leqslant H_c$, the overall zero field susceptibility of the system is a geometrical constant, independent of T; thus globally, linear response is broken. Besides, these walls cause the total ground-state energy per spin to assume a higher value than that obtained in the absence of demagnetising effects (where one would have only one domain for $H = 0$). On the whole, the free energy is a separable function of T and H.

A theoretical description of such a mixed state is by no means trivial: to represent a spatially inhomogeneous magnetic structure would in principle require an underlying lattice. Now, on the level of mean field theory, surface effects are missing. Thus one obtains a description of single domains where all sites are equivalent. However, there might be a way out of that problem: one could introduce a variable y representing the probability that a site belongs to a domain or a wall to which a given magnetisation lying in the interval $(0, m(y))$ is associated. Within mean field theory, one would therefore recover the characteristics of the inhomogeneities through a probability distribution.

A similar analysis can be performed for superconducting (type I) systems, where one gets the intermediate state (Landau 1943). (We are not concerned with type II superconductors where the wall energy is *a priori* negative.)

No mean field theory, if any, has been proposed for such systems which display an intrinsic breakdown of linear response theory with: (i) a normal part (up and down domains; normal regions), (ii) a condensed part (walls; superconducting regions). These two parts are responsible respectively for thermal and magnetic properties. In both cases, the local field in the normal part sticks to the value $H_c(T)$, to which corresponds a value $M_c(T)$ of the magnetisation.

4. Interpretation of the spin glass phase: a two-fluid model

Usually, the demagnetising effects disappear when the thermodynamic limit is considered. This is not the case for fully frustrated systems or the SK model, where the Onsager field plays a crucial role (Thouless *et al* 1977). Even though plaquettes are difficult to define in the (infinite range) SK model, it is natural to assume that overblocking takes place. Its role is lessened by thermal fluctuations and is of relatively minor importance close to T_c.

Equation (4) is also suggestive of a two-fluid model introduced in the context of superfluidity (Landau and Lifchitz 1969). In that picture, a fraction $\bar{x}(T)$ of the spins has condensed, and a fraction $1 - \bar{x}(T)$ remains normal (see figure 1). The identification of the condensed part with the overblocked plaquettes seems corroborated by the fact that $\bar{x}(0) = \frac{1}{2}$ (see § 3.1). This condensed state spans all $x\,(0 < x < \bar{x})$, so that x appears in the light of § 3.2, as the probability that a given spin has an EA order parameter lying in the interval $(0, q(x))$. With tongue in cheek, one can say that at $T = 0$, a given spin has a one-in-two chance of belonging to a domain (normal) or a to a wall (condensed). As a matter of fact, the overblocking effect enables us to define what a wall can be in such systems. Moreover, at low temperatures, the width of a wall cannot be neglected relative to the domain size (the structure is thus very ramified). With such an interpretation, the SK solution can be viewed as a solution without walls (frustrated single domain). This idea of a wall can be extended to amorphous materials where one similarly expects soft regions responsible for the low-stress response.

5. Conclusion

We have presented an interpretation of Parisi's order parameter reflecting the presence of walls or overblocked regions a natural consequence is to view a spin glass as a two-fluid system. The overblocking effect provides a natural explanation for the freezing of the magnetisation in the impure ($J_0 \neq 0$) Ising case, when one reaches the ferromagnetic-mixed phase boundary (Toulouse 1980). If our picture is correct at all, an interesting byproduct of Parisi's theory could be to provide a description, within the framework of mean field theory, of nucleation phase transitions.

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